Problem Set 9.8

(Taylor series for $f(x)$ at a) $f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$

1. Find the Taylor series for $\sin x$ at $a=\frac{\pi}{3}$.

(Maclaurin series for $f(x)$) $f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n$

2. Find the Maclaurin series for $e^x\cos x$ using the Maclaurin series for e^x and $\cos x$.

3. Find the Maclaurin series for $\dfrac{\sin x - x + \frac{x^3}{3!}}{x^5}$.

(Binomial series)

$$(1+x)^p = 1+\binom{p}{1}x+\binom{p}{2}x^2+\binom{p}{3}x^3+\cdots,$$

where p is real number and $|x|<1$.

4. Find the Maclaurin series for $\sqrt{1+x^2}$ using the Binomial series.

Problem Set 9.9

(Taylor polynomial of order n based at a)

$$P_n(x)=\sum_{k=0}^{n}\frac{f^{(k)}(a)}{k!}(x-a)^k$$

(Remainder or error for Taylor series based at a)

$$R_n(x)=\frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1},$$

where $f(x)=P_n(x)+R_n(x)$ and c is some point between x and a.

(Maclaurin polynomial of order n)

$$P_n(x)=\sum_{k=0}^{n}\frac{f^{(k)}(0)}{k!}x^k$$

5. Find the Taylor polynomial of order 2 based at $a=1$ for $f(x)=\ln(x+2)$.

6. Find the Maclaurin polynomial of order 3 for $f(x)=e^{-3x}$.

7. Consider $f(x)=\sqrt{x}$ to approximate $\sqrt{1.1}$.
(1) Find $P_2(x)$ based at $a=1$ for $f(x)$.

(2) Use $P_2(x)$ to approximate $\sqrt{1.1}$.

8. Consider $f(x)=\sin x$ to approximate $\sin 48^\circ$.
(1) Find $P_3(x)$ based at $a=\frac{\pi}{4}$ for $f(x)$.

(2) Use $P_3(x)$ to approximate $\sin 48^\circ$.

(3) Give a good bound for the error of the approximation.